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As the easiest system attainable by experiment we chose a chain 6-polar filter (Figure 1), which would imitate the properties of the first two wave types of an acoustic wave guide.

A. Main Aspects of the Theory

The equations describing stationary wave processes in such a multipolar filter are:

$$\begin{aligned} [v_1(k-1) - v_1(k)] - [v_1(k) - v_1(k+1)] &= L_1 C_1 \frac{d^2 v_1}{dt^2} + \frac{L_1}{L_0} v_1 - \frac{L_1}{L_0} v_2 \\ [v_2(k-1) - v_2(k)] - [v_2(k) - v_2(k+1)] &= L_2 C_2 \frac{d^2 v_2}{dt^2} + \frac{L_2}{L_0} v_2 - \frac{L_2}{L_0} v_1. \end{aligned} \quad (1)$$

The general solution of these equations gives two different natural waves:

$$\begin{aligned} v_1 &= A e^{i(\omega t + \varphi_1 n)} + B e^{i(\omega t - \varphi_1 n)} + C e^{i(\omega t + \varphi_2 n)} + D e^{i(\omega t - \varphi_2 n)} \\ v_2 &= k_1 A e^{i(\omega t + \varphi_1 n)} + k_1 B e^{i(\omega t - \varphi_1 n)} + k_2 C e^{i(\omega t + \varphi_2 n)} + k_2 D e^{i(\omega t - \varphi_2 n)} \end{aligned} \quad (2)$$

$n = 0, 1, 2, \dots$

which may also be called normal if we carry out the space-time analogy between waves and oscillations of connected pendulums (see C) [3].

In expression (2) each normal wave is described by two expressions, as it can be radiated in two directions.

The normal wave numbers k_1 and k_2 are found according to general formula (10), (see C) [3],

$$\theta \sin^2 \frac{\varphi_{1,2}}{2} = \left(\sin \frac{\varphi'}{2} - \sin \frac{\varphi''}{2} \right) \pm \sqrt{\left(\sin \frac{\varphi'}{2} - \sin \frac{\varphi''}{2} \right)^2 + 4\lambda_1 \lambda_2}, \quad (3)$$

and k_1 and k_2 are the coefficients of the amplitude distribution of normal waves; they are found by

$$k_{1,2} = \left(\frac{v_2}{v_1} \right)_{\varphi=\varphi_{1,2}} = \frac{1}{2\lambda_1} \left[\sin \frac{\varphi'}{2} - \sin \frac{\varphi''}{2} \mp \sqrt{\left(\sin \frac{\varphi'}{2} - \sin \frac{\varphi''}{2} \right)^2 + 4\lambda_1 \lambda_2} \right]. \quad (4)$$

Here the following notations are introduced:

$$\lambda_1 = \frac{L_1}{L_0}; \lambda_2 = \frac{L_2}{L_0}; \sin \frac{\varphi'}{2} = L_1 C_1 \omega^2 - \frac{L_1}{L_0}; \sin \frac{\varphi''}{2} = L_2 C_2 \omega^2 - \frac{L_2}{L_0}.$$

If the above system be considered as two chains arranged in parallel and consisting of the usual four-polar filters interacting on each other through the inductance L_0 , then φ' and φ'' may be called wave numbers of parallel systems (see B) [3].

The system obtained from our chains of multipolar filters by grounding one of them is a parallel system in this case. This definition of a parallel system permits a concept of coupling of the systems by analogy with Mandel'shtam's coefficient of coupling:

$$M = \frac{L_1 L_2}{L_0^2 \left(\sin \frac{\varphi'}{2} - \sin \frac{\varphi''}{2} \right)^2} \quad (5)$$

The coupling M is determined by the disturbance of the wave numbers. When M is small, the systems do not interact; when it is great, $\varphi' = \varphi''$ then they interact even with a very weak connection λ .

Assigning the amplitude distribution in the first compartment ($n=0$), in accordance with the ratios

$$\frac{v_2}{v_1} = k_1 \text{ or } \frac{v_2}{v_1} = k_2, \quad (6)$$

- 2 -
CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

we excite, respectively, waves of the first and second type, which are distinguished by the dependence of wave numbers φ_1 and φ_2 on frequency and forms of amplitude distribution. In accordance with (4), in a wave of the first type both elements of the compartment oscillate in phase; and in the second wave type, in antiphase; hence we shall call them "sympase" and "antipase." The phase velocity of propagation is less in sympase than in antipase waves.

If we take a symmetrical filter with parameters $C_1 = C_2$ and $L_1 = L_2$, these wave types will appear as algebraic expressions of the first and second wave of an acoustic wave guide.

B. Experimental Installation

Figure 2 [photograph not reproduced] shows the multipolar filter, made of honeycomb induction coils and variable condensers, adjusted with a precision correct to 0.5 percent. It was used in this experiment.

A constant electromotive force was maintained at the input of the chain by means of a push-pull generator, connected with the filter on the same principle as a stepdown autotransformer. A double-track generator outlet was used to excite antipase waves.

A basic experimental difficulty consisted in attaining running waves in a real ten-link chain in a wide-frequency range simultaneously for both wave types. This was done by applying voltage v_1 and v_2 from a high-frequency generator to the first compartment ($n = 0$), in accordance with ratio (6), and the capacities of the following 5 or 6 filter links were shunted by a series of resistances, decreasing monotonically from hundreds of thousands to hundreds of ohms at the free terminal of the chain. (Figure 3). This "wedge" of resistances resembles a medium with an attenuation with smoothly varying properties, due to which there is no reflection and the wave is completely absorbed. Its effect is similar to the effect of a wedge applied in practical wave guides. With the help of this wedge a run of the order of 95 to 97 percent was obtained in a wide frequency range for both wave types and the existence of running sympase and antipase waves was corroborated through the phase and amplitude distribution along the chains.

Figure 4 shows the phase distribution along the filter links for a running wave recorded by an oscillograph for a fixed frequency ω , and the amplitude distribution of voltages measured in the links of the filter by a VEB-type cathode voltmeter. Measures were taken for a filter with parameters $L_1 = L_2 = 145 \mu\text{H}$; $C_1 = C_2 = 165 \mu\text{F}$; $L_0 = 300 \mu\text{H}$.

As may be seen from Figure 4, the change in phase of a wave in moving along the filter, as determined oscillographically from Lissajous' figures, may serve as a rough measure of the dependence of phase velocity on frequency.

C. Dispersion of Normal Waves. Locking Phenomenon

In studying the dispersion, that is, the dependence of φ on ω , the vertical wave method gave more exact measurements than the Lissajous-figure method.

To receive vertical waves the multipolar filter was loaded with reactive resistances.

The experiment was conducted in the following way. The generator frequency was varied until the coupling was adjusted to resonance as registered by a cathode voltmeter connected with any filter.

- 3 -

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

If the terminals of the links were loaded with considerably less resistance than the wave resistance, when there was resonance in its length, an integral number of half-waves was piled up.

λ was determined by plotting the curves of voltage distribution in the link for resonance frequencies against internodal distance. From these data a graph of $\varphi(\omega)$ was made. As may be seen from Figure 5, the results obtained in a filter with the below-mentioned parameters corroborated the theoretical calculations given by formula (3).

Thanks to the discreteness of the chain, both types of waves had threshold frequencies of $\omega_{thres C} = \frac{2}{\sqrt{LC}}$ and $\omega_{thres A} = \sqrt{\frac{4}{LC} + \frac{2}{L_0 C}}$, above which no wave process exists.

The break in its propagation at frequencies lower than the critical $\omega_{thres A}$ is a specific characteristic of the antiphase wave. This phenomenon is analogous to the effect of "locking" in wave guides [3] and in acoustic tubes for wave type II. This phenomenon does not occur in symphase waves.

D. Space Pulsations

Another interesting fact which requires experimental proof was the appearance of space pulsations. If both types of running waves are excited, as a result of their superposition and thanks to the difference in phase velocity, space pulsations are generated analogously to the time pulsations of two connected pendulums.

In fact, on the basis of formula (2), provided that $\varphi' = \varphi''$, it follows from solution (1) that:

$$\begin{aligned} v_1 &= F \cos \beta n e^{i(\omega t - \alpha n)}, \\ v_2 &= F \sin \beta n e^{i(\omega t - \alpha n)}, \end{aligned}$$

where

$$\begin{aligned} 2\alpha &= \sqrt{|\varphi_1|} + \sqrt{|\varphi_2|}, \\ 2\beta &= \sqrt{|\varphi_1|} - \sqrt{|\varphi_2|}, \end{aligned} \quad (7)$$

It may, therefore, be considered that one wave travels in a chain, the energy of which swings periodically from one partial system to another.

To receive space pulsations, one of the capacities of the first compartment $n=0$, for instance C_2 , is grounded and a second capacity has impressed a sinusoidal voltage. At the outlet of the multipolar filter a wedge is placed to provide a run for both types of waves. If the coupling between the filters through the inductance L_0 tends to vanish, a wave would be propagated only in the first fractional 4-pole catenary.

Because of the coupling L_0 a swing in the energy was observed with the space frequency equal to the normal wave frequency $\varphi_2 - \varphi_1$. Pulsations were determined by plotting the voltage distribution curves in the filter.

In the identical fractional filters $C_1 = C_2$ and $L_1 = L_2$ a complete swing of energy occurred.

The table shows the theoretical and experimental dependence of space-pulsation cycles on frequency plotted for a filter of 26 links with the parameters previously given.

- 4 -

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

Table of Dependence of Space-Pulsation Cycles on Frequency

(A stage is expressed by the number of compartments)

Frequency in mertz (1,000 cycles/sec)	Theoretical	Experimental
1.1	9	10
1.2	10	11
1.32	11.5	13
1.5	12	14
1.7	12	14
1.86	10.5	12
1.95	8	10

E. Unsymmetrical Filters

Since the chain, $L_1 = L_2$ and $C_1 = C_2$, is characterized by a coherence limited in force, both waves will occur in both systems with identical amplitudes. A chain in which $C_1 \neq C_2$ and $L_1 \neq L_2$ is of course more general.

Here the method of measuring the dependence of the coefficients k on frequency consists of selecting, at a given frequency ω , an amplitude distribution at the input V_1 such that space pulsations vanish in it. In accordance with (5), this relation is equal to the distribution coefficient k . Figure 6, a shows the result of measuring $k(\omega)$ for a filter consisting of 10 links with parameters $C_1 = 100 \mu F$, $C_2 = 200 \mu F$, $L_1 = 145 \mu H$, $L_2 = 300 \mu H$, $L_0 = 300 \mu H$. Figure 6 gives two frequency areas 0 - 1.2 megacycles per second and 1.93 - 2.7 megacycles per second; in area I the symphase wave does not fade, but the antiphase wave fades, and vice versa in area II. Consequently, regardless of the excitation force in the chain, only symphase waves will occur in area I and only antiphase waves in area II in the case represented by Figure 6. The experimental curves $\varphi(\omega)$ for both waves are shown in b of Figure 6.

The case when C_1 is considerably greater or less than C_2 differs from the case where $C_1 = C_2$ and $L_1 = L_2$ also in that its coupling coefficient $M(5)$ has small values and normal wave numbers close to the corresponding fractional ones (shown in b of Figure 6 by the dotted line (see B) $\sqrt{3}$).

By selecting the parameters of a filter and the coupling coefficients, it is possible to divide the frequency axis for symphase and antiphase waves, making the "locking" frequency for antiphase waves higher than the limited frequency of a symphase wave.

In this case, regardless of the excitation force, the chain of a multipolar filter appears to be a band filter for every type of normal wave (see B) $\sqrt{3}$. This phenomenon was discovered in the ten-compartment filter described above.

The measurements demonstrated that, in the frequency interval 0-980 kilocycles per second a symphase wave occurred, an antiphase wave from 1.6 to 2.8 megacycles per second and that wave processes were absent in the interval from 980 kilocycles per second to 1.6 megacycles per second which coincides within 5 percent with the calculated data.

We take the opportunity to express our thanks to Professor P. K. Kraushin for his guidance and aid in our work.

- 5 -

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50X1-HUM

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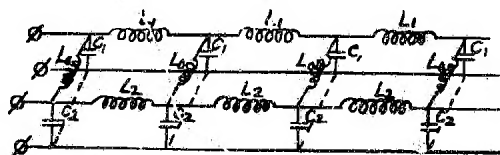


Figure 1. Diagram of a chain filter made of six poles with capacities C_1 and C_2 and inductances L_1, L_2, L_3 .

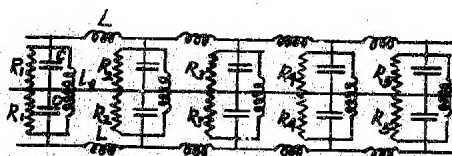


Figure 3. "Wedge" of resistance $R_1 > R_2 > R_3 > R_4 > R_5$, decreasing towards the free terminal of the chain.

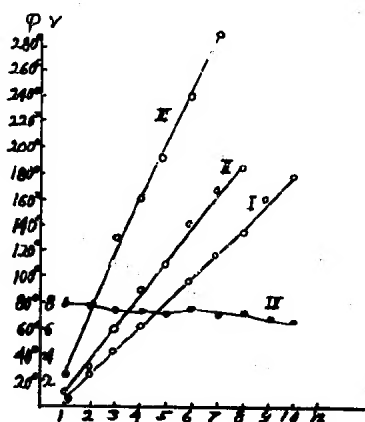


Figure 4. Phase (I, II, III) and amplitude (IV) distribution curves at filter links for a running wave.
I: $\nu = 410$ kilocycles per second; II: $\nu = 443$ kilocycles per second; III: $\nu = 720$ kilocycles per second.

- 7 -

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50X1-HUM

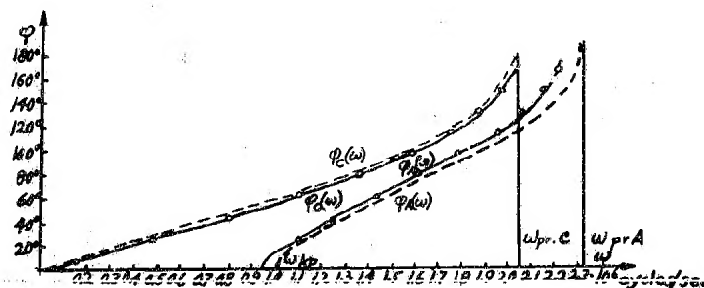


Figure 5. Curves showing the dependence of normal wave numbers ϕ_1 and ϕ_2 on frequency ω for symphase $\phi_C(\omega)$ and antiphase $\phi_A(\omega)$ waves; the experimental curve is shown by unbroken lines, the theoretical by dashes.

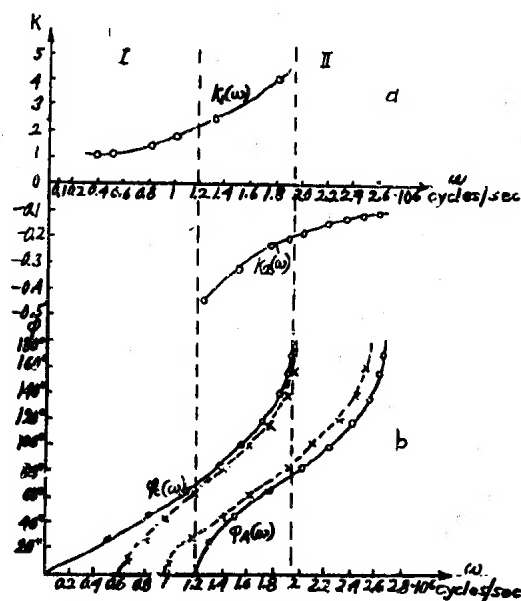


Figure 6. "a" is the experimental curve of the dependence of the coefficients of amplitude distribution of normal waves $k_1(\omega)$ and $k_2(\omega)$ on frequency; "b" gives curves $\phi_C(\omega)$ and $\phi_A(\omega)$; I is the area of antiphase wave damping; II is the area of symphase wave damping.

- E N D -

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